Building and Supporting a Highly Effective System with Mathematics

Handouts

2017 Alaska School Leadership Institute

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youcubed at Stanford University

Building a Mathematical Mindset Community

Teachers and students believe *everyone* can learn maths at HIGH LEVELS.

- Students are not tracked or grouped by achievement
- All students are offered high level work
- "I know you can do this" "I believe in you"
- Praise effort and ideas, not the person
- Students vocalize self-belief and confidence

Communication and *connections* are valued.

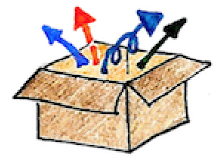
- Students work in groups sharing ideas and visuals.
- Students relate ideas to previous lessons or topics
- Students connect their ideas to their peers' ideas, visuals, and representations.
- Teachers create opportunities for students to see connections.
- Students relate ideas to events in their lives and the world.

The maths is VISUAL.

- Teachers ask students to draw their ideas
- Tasks are posed with a visual component
- Students draw for each other when they explain
- Students gesture to illustrate their thinking

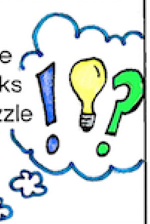
The maths is OPEN.

- Students are invited to see maths differently
- Students are encouraged to use and share different ideas, methods, and perspectives
- Creativity is valued and modeled.
- Students' work looks different from each other
- Students use ownership words - "my method", "my idea"



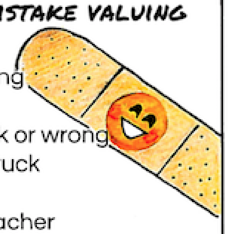
The environment is filled with *WONDER* and *CURIOSITY.*

- Students extend their work and investigate
- Teacher invites curiosity when posing tasks
- Students see maths as an unexplored puzzle
- Students freely ask and pose questions
- Students seek important information
- "I've never thought of it like that before."



The classroom is a risk-taking, *MISTAKE VALUING* environment

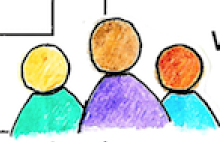
- Students share ideas even when they are wrong
- Peers seek to understand rather than correct
- Students feel comfortable when they are stuck or wrong
- Teachers and students work together when stuck
- Tasks are low floor/high ceiling
- Students disagree with each other and the teacher

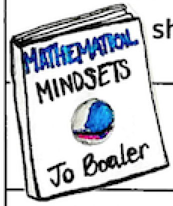


Developed by Jo Boaler/Youcubed.org and Tulare County Office of Education

Recommendations for Task/Lesson Design

- Open the task to encourage multiple methods, pathways and representations.
- Pose a problem before teaching the method.
- Design a task that allows all learners to contribute to the learning and have room for extension.
- Make opportunities for students to authentically share their thinking with peers.
- Add a visual component.
- Add the requirement to convince and reason, be skeptical.





Powerful Questions to develop a deep level of understanding

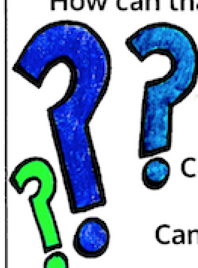
How do you see that idea?

Why does that answer make sense?

Why does that method work?

How is that method connected to others?

How can that idea be represented in different ways?



Can you prove it?

Can you prove it visually?

Can you justify your thinking?

Can you predict what would happen if....?

Did you make any interesting mistakes?

Developed by Jo Boaler/Youcubed.org and Tulare County Office of Education youcubed at Stanford University

Positive Norms to Encourage in Math Class

By Jo Boaler

1. Everyone Can Learn Math to the Highest Levels.
Encourage students to believe in themselves. There is no such thing as a "math" person. Everyone can reach the highest levels they want to, with hard work.

2. Mistakes are Valuable
Mistakes grow your brain! It is good to struggle and make mistakes.

3. Questions are Really Important
Always ask questions, always answer questions. Ask yourself: why does that make sense?

4. Math is about Creativity and Making Sense
Math is a very creative subject that is, at its core, about visualizing patterns and creating solution paths that others can see, discuss and critique.

5. Math is about Connections and Communicating
Math is a connected subject, and a form of communication. Represent math in different forms eg words, a picture, a graph, an equation, and link them. Color code!

6. Depth is much more Important than Speed
Top mathematicians, such as Laurent Schwartz, think slowly and deeply.

7. Math Class is about Learning not Performing
Math is a growth subject, it takes time to learn and it is all about effort.

Required Fluencies in K-6

Grade	Standard	Fluency
K	K.OA	Add/subtract up to 5
1	1.OA.6	Add/subtract up to 10
2	2.OA.2 2.NBT.5	Add/subtract up to 20 (know single-digit sums from memory Add/subtract up to 100
3	3.OA.7 3.NBT.2	Multiply/divide up to 100 (know single-digit products from memory Add/subtract up to 1000
4	4.NBT.4	Add/subtract up to 1,000,000
5	5.NBT.5	Multi-digit multiplication
6	6.NS.2 6.NS.3	Multi-digit division Multi-digit decimal operations

Domains

<table>
<tr><th colspan="11">Standards for Mathematical Content</th></tr>
<tr><th>Kindergarten</th><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>High School</th><th rowspan="8">Conceptual Categories
Modeling</th></tr>
<tr><td colspan="2">Counting and Cardinality</td><td colspan="7"></td><td rowspan="3">Number & Quantity</td></tr>
<tr><td colspan="6">Number and Operations in Base Ten</td><td colspan="2">Ratios and Proportional Relationships</td><td></td></tr>
<tr><td colspan="3"></td><td colspan="3">Number and Operations - Fractions</td><td colspan="3">Number System</td></tr>
<tr><td colspan="6" rowspan="2">Operations and Algebraic Thinking</td><td colspan="3">Expressions and Equations</td><td>Algebra</td></tr>
<tr><td colspan="2"></td><td>Functions</td><td>Functions</td></tr>
<tr><td colspan="9">Geometry</td><td>Geometry</td></tr>
<tr><td colspan="6">Measurement and Data</td><td colspan="3">Statistics and Probability</td><td>Statistics and Probability</td></tr>
</table>

Domains are large groups of related standards. Each shaded row shows how domains at the earlier grades progress and lead to conceptual categories at the high school levels. The right side of the chart lists the five **conceptual categories** for high school. Selecting one conceptual category and moving left along the row shows the domains at the middle and elementary school levels from which this concept builds. Modeling, the sixth conceptual category, is incorporated throughout the other five high school categories.

Overall, the progressions of the standards begin and end in different grades, avoiding the re-teaching of concepts that should have been mastered. This allows for higher rigor overall, which is key to laying the foundation for high school mathematics standards and college/career preparedness.

For each of the grade-spans (K-2, 3-5, 6-8, and 9-12) an overview of the topics to be covered follows.

Engaging in the Mathematical Practices (Look-fors)

Mathematics Practices		Students:	Teachers:
Overarching habits of mind of a productive math thinker	**1. Make sense of problems and persevere in solving them**	☐ Understand the meaning of the problem and look for entry points to its solution ☐ Analyze information (givens, constrains, relationships, goals) ☐ Make conjectures and plan a solution pathway ☐ Monitor and evaluate the progress and change course as necessary ☐ Check answers to problems and ask, "Does this make sense?" Comments:	☐ Involve students in rich problem-based tasks that encourage them to persevere in order to reach a solution ☐ Provide opportunities for students to solve problems that have multiple solutions ☐ Encourage students to represent their thinking while problem solving Comments:
	6. Attend to precision	☐ Communicate precisely using clear definitions ☐ State the meaning of symbols, carefully specifying units of measure, and providing accurate labels ☐ Calculate accurately and efficiently, expressing numerical answers with a degree of precision ☐ Provide carefully formulated explanations ☐ Label accurately when measuring and graphing Comments:	☐ Emphasize the importance of precise communication by encouraging students to focus on clarity of the definitions, notation, and vocabulary used to convey their reasoning ☐ Encourage accuracy and efficiency in computation and problem-based solutions, expressing numerical answers, data, and/or measurements with a degree of precision appropriate for the context of the problem Comments:
Reasoning and Explaining	**2. Reason abstractly and quantitatively**	☐ Make sense of quantities and relationships in problem situations ☐ Represent abstract situations symbolically and understand the meaning of quantities ☐ Create a coherent representation of the problem at hand ☐ Consider the units involved ☐ Flexibly use properties of operations Comments:	☐ Facilitate opportunities for students to discuss or use representations to make sense of quantities and their relationships ☐ Encourage the flexible use of properties of operations, objects, and solution strategies when solving problems ☐ Provide opportunities for students to decontextualize (abstract a situation) and/or contextualize (identify referents for symbols involved) the mathematics they are learning Comments:
	3. Construct viable arguments and critique the reasoning of others	☐ Use definitions and previously established causes/effects (results) in constructing arguments ☐ Make conjectures and use counterexamples to build a logical progression of statements to explore and support ideas ☐ Communicate and defend mathematical reasoning using objects, drawings, diagrams, and/or actions ☐ Listen to or read the arguments of others ☐ Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments Comments:	☐ Provide and orchestrate opportunities for students to listen to the solution strategies of others, discuss alternative solutions, and defend their ideas ☐ Ask higher-order questions which encourage students to defend their ideas ☐ Provide prompts that encourage students to think critically about the mathematics they are learning Comments:

 * All indicators are not necessary for providing full evidence of practice(s). Each practice may not be evident during every lesson.

Mathematics Practices		Students:	Teacher(s):
Modeling and Using Tools	**4. Model with mathematics**	☐ Apply prior knowledge to solve real world problems ☐ Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and/or formulas ☐ Use assumptions and approximations to make a problem simpler ☐ Check to see if an answer makes sense within the context of a situation and change a model when necessary Comments:	☐ Use mathematical models appropriate for the focus of the lesson ☐ Encourage student use of developmentally and content-appropriate mathematical models (e.g., variables, equations, coordinate grids) ☐ Remind students that a mathematical model used to represent a problem's solution is 'a work in progress,' and may be revised as needed Comments:
	5. Use appropriate tools strategically	☐ Make sound decisions about the use of specific tools (Examples might include: calculator, concrete models, digital technologies, pencil/paper, ruler, compass, protractor) ☐ Use technological tools to visualize the results of assumptions, explore consequences, and compare predications with data ☐ Identify relevant external math resources (digital content on a website) and use them to pose or solve problems ☐ Use technological tools to explore and deepen understanding of concepts Comments:	☐ Use appropriate physical and/or digital tools to represent, explore and deepen student understanding ☐ Help students make sound decisions concerning the use of specific tools appropriate for the grade level and content focus of the lesson ☐ Provide access to materials, models, tools and/or technology-based resources that assist students in making conjectures necessary for solving problems Comments:
Seeing structure and generalizing	**7. Look for and make use of structure**	☐ Look for patterns or structure, recognizing that quantities can be represented in different ways ☐ Recognize the significance in concepts and models and use the patterns or structure for solving related problems ☐ View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems Comments:	☐ Engage students in discussions emphasizing relationships between particular topics within a content domain or across content domains ☐ Recognize that they quantitative relationships modeled by operations and their properties remain important regardless of the operational focus of a lesson ☐ Provide activities in which students demonstrate their flexibility in representing mathematics in a number of ways e.g., 76 = (7 x 10) + 6; discussing types of quadrilaterals, etc. Comments:
	8. Look for and express regularity in repeated reasoning	☐ Notice repeated calculations and look for general methods and shortcuts ☐ Continually evaluate the reasonableness of intermediate results (comparing estimates), while attending to details, and make generalizations based on findings Comments:	☐ Engage students in discussion related to repeated reasoning that may occur in a problem's solution ☐ Draw attention to the prerequisite steps necessary to consider when solving a problem ☐ Urge students to continually evaluate the reasonableness of their results Comments:

APPENDIX D. EIGHT EFFECTIVE MATHEMATICS TEACHING PRACTICES

Teaching Practice	Purpose	What the Teacher Does	What the Students Do
1. Establish mathematics goals to focus learning.	• Set the stage to guide instructional decisions. • Expect students to understand the purpose of a lesson beyond simply repeating the standard.	Considers broad goals as well as the goals of the unit and the actual lesson, including the following: • What is to be learned? • Why is the goal important? • Where do students need to go? • How can learning be extended?	• Make sense of new concepts and skills, including connections to concepts learned in previous grades. • Experience connections among the standards and across domains. • Deepen their understanding and expect mathematics to make sense.
2. Implement tasks that promote reasoning and problem solving.	• Provide opportunities for students to engage in exploration and make sense of important mathematics. • Encourage students to use procedures in ways that are connected to understanding.	Chooses tasks that • are built on current student understandings. • have various entry points with multiple ways for the problems to be solved. • are interesting to students.	• Work to make sense out of the task and persevere in solving problems. • Use a variety of models and materials to make sense of the mathematics in the task. • Convince themselves and others the answer is reasonable.
3. Use and connect mathematical representations.	• Provide concrete representations that lead students to develop conceptual understanding and later connect that understanding to procedural skills. • Provide a variety of representations that range from using physical models to using abstract notations.	• Uses tasks that allow students to use a variety of representations. • Encourages the use of different representations, including concrete models, pictures, words, and numbers, that support students in explaining their thinking and reasoning.	• Use materials to make sense out of problem situations. • Connect representations to mathematical ideas and the structure of big ideas, including operational sense with whole numbers, fractions, and decimals.
4. Facilitate meaningful mathematical discourse.	• Provide students with opportunities to share ideas, clarify their understanding, and develop convincing arguments. • Advance the mathematical thinking of the whole class by talking and sharing aloud.	• Engages students in explaining their mathematical reasoning in small group and classroom situations. • Facilitates discussions among students that support making sense of a variety of strategies and approaches. • Scaffolds classroom discussions so that connections between representations and mathematical ideas take place.	• Explain their ideas and reasoning in small groups and with the entire class. • Listen to the reasoning of others. • Ask questions of others to make sense of their ideas.

Teaching Practice	Purpose	What the Teacher Does	What the Students Do
5. Pose purposeful questions.	• Reveal students' current understanding of a concept. • Encourage students to explain, elaborate, and clarify their thinking. • Make the learning of mathematics more visible and accessible for students.	• Asks questions that build on and extend student thinking. • Is intentional about the kinds of questions asked to make the mathematics more visible to students. • Uses wait time to provide students with time to think and examine their ideas.	• Think more deeply about the process of the mathematics rather than simply focusing on the answer. • Listen to and comment on the explanations of others in the class.
6. Build procedural fluency from conceptual understanding.	• Provide experiences with concrete materials that allow students to make sense of important mathematics and flexibly choose from a variety of methods to solve problems.	• Provides opportunities for students to reason about mathematical ideas. • Expects students to explain why their strategies work. • Connects student methods to efficient procedures as appropriate.	• Understand and explain the procedures they are using and why they work. • Use a variety of strategies to solve problems and make sense of mathematical ideas. • Do not rely on shortcuts or tricks to do mathematics.
7. Support productive struggle in learning mathematics.	• Provide opportunities for productive struggle, which is significant and essential to learning mathematics with understanding. • Allow students to grapple with ideas and relationships. • Give students ample time to work with and make sense of new ideas, which is critical to their learning with understanding.	• Supports student struggle without showing and telling a procedure but rather focusing on the important mathematical ideas. • Asks questions that scaffold and advance student thinking. • Builds questions and plans lessons based on important student misconceptions rather than focusing on the correct answer. • Recognizes the importance of effort as students work to make sense of new ideas.	• Stick to a task and recognize that struggle is part of making sense. • Ask questions that will help them to better understand the task. • Support each other with ideas rather than telling others the answer or how to solve a problem.
8. Elicit and use evidence of student thinking.	• Elicit and use evidence of student thinking, which helps teachers access learning progress and can be used to make instructional decisions during the lessons as well as help to prepare what will occur in the next lesson. • Assess student thinking and understanding by using formative assessment through student written and oral ideas.	• Determines what to look for in gathering evidence of student learning. • Poses questions and answers student questions that provide information about student understanding, strategies, and reasoning. • Uses evidence to determine next steps of instruction.	• Accept that reasoning and understanding are as important as the answer to a problem. • Use mistakes and misconceptions to rethink their understanding. • Ask questions of the teacher and peers to clarify confusion or misunderstanding. • Assess progress toward developing mathematical understanding.

Source: Adapted from National Council of Teachers of Mathematics (2014).

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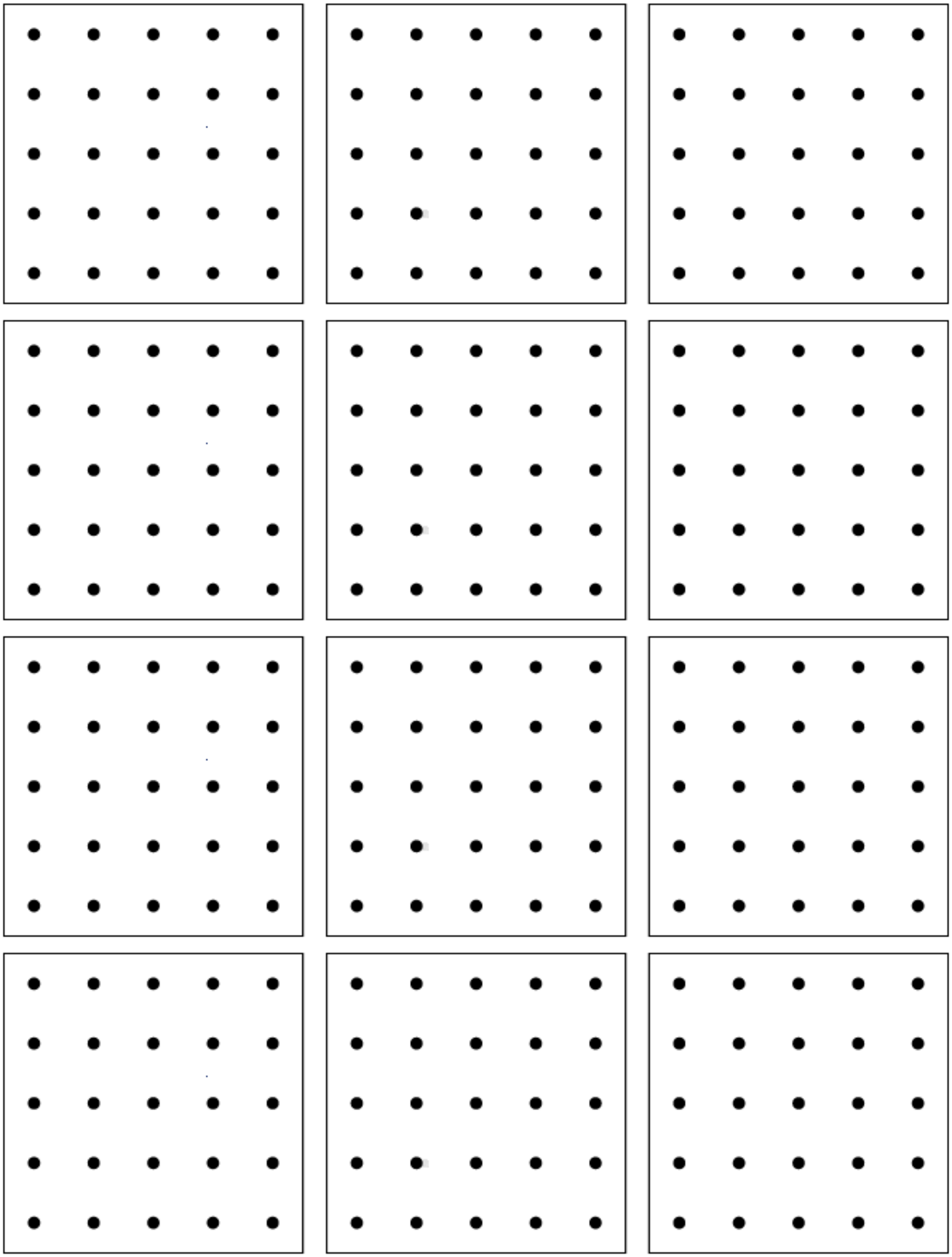
Name____



Guiding Principles for School Mathematics

Full statements of the Guiding Principles follow; *Principles to Actions* elaborates the unique importance of each, as summarized briefly below each statement. The first Guiding Principle, Teaching and Learning, has primacy among the Guiding Principles, with the others serving as the Essential Elements that support it.

Teaching and Learning. *An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.*

The teaching of mathematics is complex. It requires teachers to have a deep understanding of the mathematical content that they are expected to teach and a clear view of how student learning of that mathematics develops and progresses across grades. It also calls for teachers to be skilled at using instructional practices that are effective in developing mathematics learning for all students. The eight Mathematics Teaching Practices (see fig. 1) describe the essential teaching skills derived from the research-based learning principles, as well as other knowledge of mathematics teaching that has emerged over the last two decades.

Access and Equity. *An excellent mathematics program requires that all students have access to a high-quality mathematics curriculum, effective teaching and learning, high expectations, and the support and resources needed to maximize their learning potential.*

Equitable access means high expectations, adequate time, consistent opportunities to learn, and strong support that enable students to be mathematically successful. Instead of one-size-fits-all practices and the differential expectations for students who are placed in different academic tracks, equitable access means accommodating differences to meet a common goal of high levels of learning by all students.

Curriculum. *An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world.*

A robust curriculum is more than a collection of activities; instead, it is a coherent sequencing of core mathematical ideas that are well articulated across the grades. Such an effective curriculum incorporates problems in contexts from everyday life and other subjects whenever possible. These tasks engage students and generate interest and curiosity in the topics under investigation.

Tools and Technology. *An excellent mathematics program integrates the use of mathematical tools and technology as essential resources to help students learn and make sense of mathematical ideas, reason mathematically, and communicate their mathematical thinking.*

Available tools and technology help teachers and students visualize and concretize mathematics abstractions, and when these resources are used appropriately, they support effective teaching and meaningful learning.

Assessment. *An excellent mathematics program ensures that assessment is an integral part of instruction, provides evidence of proficiency with important mathematics content and practices, includes a variety of strategies and data sources, and informs feedback to students, instructional decisions, and program improvement.*

Effective assessment supports and enhances the learning of important mathematics by furnishing useful formative and summative information to both teachers and students. Productive mathematics assessment is a process that is coherently aligned with learning goals and makes deliberate use of the data gathered as evidence of learning and provides guidance for next instructional steps and programmatic decision making. Students learn to assess and recognize high quality in their own work.

Professionalism. *In an excellent mathematics program, educators hold themselves and their colleagues accountable for the mathematical success of every student and for personal and collective professional growth toward effective teaching and learning of mathematics.*

Effective schools communicate a tangible sense of the professional imperative to grow personally and collectively and to hold one another accountable for this growth. Professionals who are responsible for students' mathematics learning are never satisfied with their accomplishments and are always working to increase the impact that they have on their students' mathematics learning. Moreover, they cultivate and support a culture of professional collaboration and continual improvement that is driven by an abiding sense of interdependence and collective responsibility.

Actions

Although principles provide guidance and structure, actions determine impact. *Principles to Actions* argues that ensuring mathematical success for all will take **teachers** who, among other actions—

- plan and implement effective instruction as described by the Mathematics Teaching Practices;
- develop socially, emotionally, and academically safe environments for mathematics teaching and learning—environments in which students feel secure and confident in engaging with one another and with teachers;
- evaluate curricular materials and resources to determine the extent to which these materials align with the standards, ensure coherent development of topics within and across grades, promote the mathematical practices, and support effective instruction that implements the Mathematics Teaching Practices;
- incorporate mathematical tools and technology as an everyday part of the mathematics classroom, recognizing that students should experience "mathematical action technologies" and physical or virtual manipulatives to explore important mathematics;
- provide students with descriptive, accurate, and timely feedback on assessments, including strengths, weaknesses, and next steps for progress toward the learning targets;
- work collaboratively with colleagues to plan instruction, solve common challenges, and provide mutual support as they take collective responsibility for student learning.

Principles to Actions argues that ensuring mathematical success for all will take **principals, coaches, specialists, and other school leaders** who, among other actions—

- make the eight Mathematics Teaching Practices a schoolwide focus that is expected for all teachers to strengthen learning and teaching for all students, and provide professional development, training, and coaching to make the implementation of these practices a priority;
- maintain a schoolwide culture with high expectations and a growth mindset;

- allocate time for teachers to collaborate in professional learning communities;
- support improvement with multifaceted assessments used to monitor progress and inform changes to instruction;
- make the mathematical success of every student a nonnegotiable priority.

Principles to Actions argues that ensuring mathematical success for all will take **leaders and policymakers in districts, states or provinces, including commissioners, superintendents and other central office administrators**, who, among other actions—

- make ongoing professional development that supports the implementation of the eight Mathematics Teaching Practices as a priority;
- allocate resources to ensure that all students are provided with an appropriate amount of instructional time to maximize their learning potential;
- eliminate the tracking of low-achieving students and instead structure interventions that provide high-quality instruction and other classroom support, such as math coaches and specialists;
- understand the devastating impact of professional isolation and create collaborative structures to maximize professional growth;
- Support risk taking and encourage new approaches that advance student learning.

Only when these words become actions and the actions lead to more productive beliefs, new norms of instructional practice, and implementation of the essential supporting elements will we overcome the obstacles that currently prevent school mathematics from ensuring success for all students.
